Interactive Supplementary Lesson Four for Grade 12 Students

General Instructions: Choose the correct answer for the following questions, which are designed for your interactive supplementary practice. The questions, correct answers, and distractive choices are linked. Attempt to answer them, and then check your response by clicking on the alternatives, which will lead you to feedback. You may attempt each question three times until you get it correct. Every time, you get it wrong, you should click on 'Try Again' to return to the same question.

1. Find all points where the function $f(x) = \sqrt{x^2 + 4x}$ has a vertical tangent line.
   A. (0, 4) and (-4, 0)
   B. (0, 0) and (-4, 0)
   C. (-4, 4) and (0, 0)
   D. (4, 4) and (0. 0)
2. If the radius of a circle is shrinking at a rate of 2 cm/sec, find the rate at which the area of the circle is decreasing at the instant when the radius is 5 cm.
   A. - 20 πcm^2/sec
   B. - 20 cm^2/sec
   C. - 62.8 cm^2/sec
   D. 7 πcm^2/sec
3. A gardener has 180 meters of fencing to enclose a rectangular garden that borders a wall. (No fencing is needed along the wall). What are the dimensions of the garden that will maximize the enclosed area?
   A. 45 m by 60 m.
   B. 90 m by 180 m.
   C. 45 m by 90 m.
   D. 60 m by 90 m.
4. Ethiopian Airlines determines that the marginal profit resulting from selling x tickets on a journey from Addis Ababa to London in hundreds of dollars, is given by P'(x) = 5 - $\frac{1}{2}$ x$^{1/3}$. Find the total profit when 27 tickets are sold.
   A. $104.65
   B. $81,000.00
   C. $104,625.00
   D. $10,462.50.
5. To evaluate the value the definite integral $\int_{-2}^{3}(x+2)dx$.
   A. 13.5
   B. 12.5
   C. -6
   D. 13

IncorrectAnsToQ1. Try Again.
IncorrectAnsToQ2. Try Again.
IncorrectAnsToQ3. Try Again.
IncorrectAnsToQ4. Try Again.
IncorrectAnsToQ5. Try Again.

Feedback

Good job. Have a look at the explanation for #Q1 below if you like and then go to the next question.

Solution:

To find the points at which the function $f(x) = \sqrt{x^2 + 4x}$ has vertical tangent lines, we need to find where the derivative of the function is undefined or goes to infinity.

First, let's find the first derivative of $f(x) = \sqrt{x^2 + 4x}$

We use the chain rule to differentiate $f(x)$. Let = $u = x^2 + 4x$, then $f(x) = \sqrt{u^2}$

Applying the chain rule: $f'(x) = \frac{d}{dx}u^{1/2} = \frac{1}{2}u^{-1/2}. \frac{du}{dx}$

Next, we need to compute $\frac{du}{dx}$:

$$u = x^2 + 4x$$

$$\frac{du}{dx}: = x^2 + 4x$$

Putting it all together, we have:

$f'(x) = \frac{1}{2}(x^2 + 4x)^{-1/2}. (2x + 4)$

Simplifying further:

$f'(x) = \frac{2(X+2)}{2\sqrt{x^2+4x}} = \frac{X+2}{\sqrt{x^2+4x}}$

A vertical tangent occurs where this derivative is undefined. The derivative $f'(x)$ will be undefined if the denominator is zero, i.e., if:

$\sqrt{x^2 + 4x} = 0$. So, we need to solve: $x^2 + 4x = 0$

Factor the quadratic expression: $x(x + 4) = 0$

The solutions are: $x = 0$ or $x = -4$

Next, we must check if these points result in the function $f(x)$ being defined:

For $x = 0$:

$f(0) = \sqrt{0^2 + 4 . 0} = \sqrt{0^2} = 0$ and so x = is a valid point.

For $x = -4$:

$f(-4) = \sqrt{(-4)^2 + 4 . (-4)} = \sqrt{16 - 16}) = \sqrt{0} = 0$, hence, $x = -4$ is also a valid point.

Conclusion

The function $f(x) = \sqrt{x^2 + 4x}$ has vertical tangent lines at the points:

- (0, 0) and (-4, 0)

Good job. Have a look at the explanation for #Q2 below if you like and then go to the next question.

Solution:

To find the rate at which the area of a circle is decreasing as its radius shrinks, we can follow these steps:

Given: The radius r is shrinking at a rate of $\frac{dr}{dt}$ = -2 cm/sec (the negative sign indicates that the radius is decreasing and the radius r = 5 cm.

Step 1: Area A of a circle is given by the formula: A = πr^2

Step 2: Differentiate with Respect to Time

We need to differentiate the area A with respect to time t: $\frac{dA}{dt} = \frac{d}{dt}(\pi r^2) = 2\pi r)\frac{dr}{dt}$

Step 3: Substitute Known Values

Now we substitute the known values into the equation. We have r = 5 cm and $\frac{dr}{dt}$ = -2 cm/sec: $\frac{dA}{dt} = 2\pi(5)(-2)$

Calculating that $\frac{dA}{dt} = 2\pi(5)(-2)$ = -20 πcm^2/sec

Conclusion

The rate at which the area of the circle is decreasing when the radius is 5 cm is:

$\frac{dA}{dt}$ = --20 πcm^2/sec

Numerically, this is approximately:

$\frac{dA}{dt}$ = -20 X 3.14 ≈ 62.8 cm^2/sec

Thus, the area is decreasing at a rate of approximately 62.8 cm^2/sec at that instant.

Good job. Have a look at the explanation for #Q3 below if you like and then go to the next question.

Solution:

To find the dimensions of the rectangular garden that will maximize the enclosed area, we can follow similar steps as in the previous problem.

Step 1: Define the Variables

Let:

- x = the width of the garden (the sides perpendicular to the wall)
- y = the length of the garden (the side parallel to the wall)

Step 2: Write the Perimeter Constraint

Since the wall does not require fencing along one side, the total length of fencing used is given by: $x + y + x = 180$

This simplifies to: $2x + y = 180$

Step 3: Express Area in Terms of One Variable

The area A of the rectangle is given by: $A = x \cdot y$

From the perimeter constraint, we can express y in terms of x:

$y = 180 - 2x$

Now substitute this expression for y back into the area formula:

$A = x(180 - 2x) = 180x - 2x^2$

Step 4: Find the Maximum Area

To maximize the area, we take the derivative of A with respect to x and set it to zero:

$\frac{dA}{dx} = 180 - 4x$

Setting the derivative equal to zero:

$180 - 4x = 0$

Solving for x:

$4x = 180 \Rightarrow x = 45$

Step 5: Find the Corresponding Length y

Substituting $x = 45$ back into the equation for y:

$y = 180 - 2(45) = 180 - 90 = 90$

Conclusion

Thus, the dimensions of the garden that will maximize the enclosed area are:

- Width $x = 45$ m
- Length $y = 90$ m

∴The dimensions of the rectangular garden that maximize the area, while using 180 m of fencing along the wall, are 45 m by 90 m.

Good job. Have a look at the explanation for #Q4 below if you like and then go to the next question.

Solution:
Let's solve the given problem where the marginal profit function is:
$P'(x) = 5 - \frac{1}{2}x^{1/3}$.
We are asked to find the total profit when 27 tickets are sold.
Step 1: Integrate the marginal profit function
To find the total profit function P(x), we integrate the marginal profit function:
$P(x) = \int(5 - \frac{1}{2}x^{1/3})\, dx$
Break this into two parts:
$P(x) = \int 5dx - \int \frac{1}{2}x^{1/3}dx$
1. The integral of (5, dx) is straightforward:
$\int 5dx = 5x$
2. For $(\int X^{1/3}$, dx), apply the power rule for integration:
$\int X^{1/3}dx = \frac{3}{4}x^{4/3}$
Thus, we have:
$\int \frac{1}{2}x^{1/3}dx = \frac{1}{2} \times \frac{3}{4}x^{4/3} = \frac{3}{8}x^{4/3}$
Step 2: Write the total profit function
Now, combining the results from the integrals, the total profit function is:
$P(x) = 5x - \frac{3}{8}x^{4/3} + C$
Where C is the constant of integration.
Step 3: Determine the constant of integration
Assume that when no tickets are sold (i.e., x = 0, the profit is zero: P(0) = 0
Substitute x = 0 into the total profit function: $0 = 5x - \frac{3}{8}0^{4/3} + C \Rightarrow C = 0$
Thus, the total profit function is: $P(x) = 5x - \frac{3}{8}x^{4/3}$
Step 4: Find the total profit when 27 tickets are sold
Now, substitute x = 27 into the profit function:
$P(27) = 5(27) - \frac{3}{8}(27)^{4/3}$
First, calculate $27^{4/3}$:
$27^{4/3} = (27^{1/3})^4 = 3^4 = 81$
Now substitute this into the equation: $P(27) = 5(27) - \frac{3}{8} \times 81$
$P(27) = 135 - \frac{243}{8}$
$P(27) = 135 - 30.375$
$P(27) = 104.625$
Final Answer:
The total profit when 27 tickets are sold is approximately 104.625 hundred dollars, or $10,462.50.

Good job. Have a look at the explanation for #Q5 below if you like.

Solution

To evaluate value the definite integral $\int_{-2}^{3}(x+2)dx$, we need to find the antiderivative of the integrand $x+2$ and then apply the limits of integration.

First, let's find the antiderivative of $x+2$:

$$\int (x+2)dx$$

Integrating term-by-term:

$$\int xdx + \int 2dx$$

The antiderivative of x is $\frac{x^2}{2}$ and the antiderivative of 2 is $2x$. Therefore:

$$\int (x+2)dx = \frac{x^2}{2} + 2x + C$$

Now, we evaluate this antiderivative from x = -2 to x = 3:

$$\left[\frac{x^2}{2} + 2x\right]_{-2}^{3}$$

First, we evaluate at the upper limit x = 3:

$$\left(\frac{3^2}{2} + 2 \cdot 3\right) = \left(\frac{9}{2} + 6\right) = \left(\frac{9}{2} + \frac{12}{2}\right) = \frac{21}{2}$$

Next, we evaluate at the lower limit x = -2:

$$\left(\frac{(-2)^2}{2} + 2 \cdot (-2)\right) = \left(\frac{4}{2} - 4\right) = (2-4) = -2$$

Now, subtract the value at the lower limit from the value at the upper limit

$$\frac{21}{2} - (-2) = \frac{21}{2} + 2 = \frac{21}{2} + \frac{4}{2} = \frac{25}{2} = 12.5.$$